

# Self-Assessment Question Set

## Quantum Physics Corner

These questions are designed to help you assess how well you understand basic techniques from calculus and complex numbers. If you can tackle them without much trouble, it's a good sign that you're ready for the course *Quantum Mechanics I: Elementary Concepts*.

### Questions

Differentiate the following functions:

1.

$$f(t) = t \sin 2^t,$$

2.

$$g(x) = \sqrt{\ln x + 1} + \ln(\sqrt{x} + 1),$$

3.

$$h(x) = e^{\sin^2 x},$$

4.

$$u(t) = t^n a^{-t^2},$$

where  $a > 0$ , and  $n$  is an integer,

5.

$$y(x) = \left(1 + \frac{1}{x}\right)^x.$$

Find  $\frac{\partial f}{\partial x}$  and  $\frac{\partial f}{\partial y}$ , if

6.

$$f(x, y) = \frac{x}{\sqrt{x^2 + y^2}},$$

7.

$$f(x, y) = e^{-a(x - \sin y)^2 + bx},$$

where  $a$  and  $b$  are some constants.

Find the following integral:

8.

$$\int_0^1 x(x-1)^{10} dx.$$

Using the method of integration by parts, find the following indefinite integrals:

9. 
$$\int x \ln x \, dx ,$$

10. 
$$\int e^x \cos x \, dx .$$

Given that  $\int_{-\infty}^{+\infty} e^{-x^2} dx = \sqrt{\pi}$ , find the following integrals:

11. 
$$\int_{-\infty}^{+\infty} x^2 e^{-x^2} dx ,$$

12. 
$$\int_{-\infty}^{+\infty} x e^{-x^2+4x} dx .$$

Let  $i$  denote the complex unit ( $i^2 = -1$ ). Find  $\operatorname{Re} z$ ,  $\operatorname{Im} z$ , and  $|z|$ , if

13. 
$$z = i^7 + i + 1 ,$$

14. 
$$z = (1 + i)^7 ,$$

15. 
$$z = \frac{1}{1+i} - \frac{1}{(1-i)^2} ,$$

16. 
$$z = \frac{a+ib}{a-ib} ,$$

17. 
$$z = e^{ia} + 2e^{ib} ,$$

where  $a$  and  $b$  are some real numbers.

Find the functions  $y(x)$  satisfying the following differential equations:

18. 
$$y'' - 4y = 0 \quad \text{with} \quad y(0) = 2 , \quad y(\pi/4) = -1 ,$$

19. 
$$y'' - 16y = 0 \quad \text{with} \quad y(0) = 3 , \quad y'(0) = 4i ,$$

20. 
$$y'' + 2y = 0 \quad \text{with} \quad y(0) = 0 , \quad y'(0) = 2\sqrt{2} .$$

## Answers

1.  $\sin 2^t + (\ln 2)2^t t \cos 2^t$
2.  $\frac{1}{2x\sqrt{\ln x+1}} + \frac{1}{2(x+\sqrt{x})}$
3.  $e^{\sin^2 x} \sin 2x$
4.  $t^{n-1} a^{-t^2} (n - 2t^2 \ln a)$
5.  $(1 + \frac{1}{x})^x [\ln(1 + \frac{1}{x}) - \frac{1}{1+x}]$
6.  $\frac{\partial f}{\partial x} = \frac{y^2}{(x^2+y^2)^{3/2}}, \quad \frac{\partial f}{\partial y} = -\frac{xy}{(x^2+y^2)^{3/2}}$
7.  $\frac{\partial f}{\partial x} = [-2a(x - \sin y) + b] e^{-a(x-\sin y)^2+bx}, \quad \frac{\partial f}{\partial y} = 2a \cos y(x - \sin y) e^{-a(x-\sin y)^2+bx}$
8.  $\frac{1}{132}$
9.  $\frac{x^2}{2} (\ln x - \frac{1}{2})$
10.  $\frac{e^x}{2} (\cos x + \sin x)$
11.  $\frac{\sqrt{\pi}}{2}$
12.  $2e^4 \sqrt{\pi}$
13.  $z = 1$
14.  $\operatorname{Re} z = 8, \quad \operatorname{Im} z = -8, \quad |z| = 8\sqrt{2}$
15.  $\operatorname{Re} z = \frac{1}{2}, \quad \operatorname{Im} z = -1, \quad |z| = \frac{\sqrt{5}}{2}$
16.  $\operatorname{Re} z = \frac{a^2-b^2}{a^2+b^2}, \quad \operatorname{Im} z = \frac{2ab}{a^2+b^2}, \quad |z| = 1$
17.  $\operatorname{Re} z = \cos a + 2 \cos b, \quad \operatorname{Im} z = \sin a + 2 \sin b, \quad |z| = \sqrt{5 + 4 \cos(a - b)}$
18.  $y(x) = -\sin 2x$
19.  $y(x) = 2e^{4ix} + e^{-4ix}$
20.  $y(x) = 2 \sinh \sqrt{2}x$